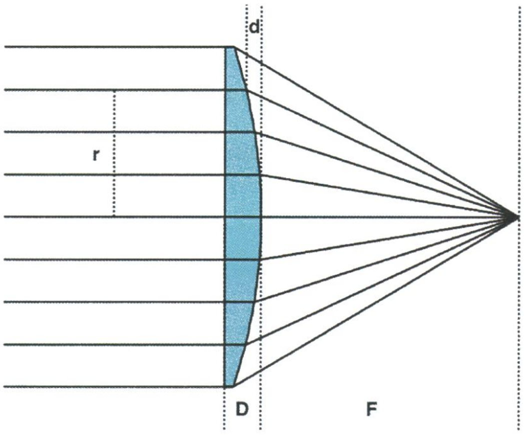
Exam : Waves & Optics

30 January 2017, 9:00-12:00

- Put your name and student number on each answer sheet.
- Answer all questions short and to the point, but complete; write legible.
- Answers that require a unit, but do not have one or the wrong one, are consider incorrect!
- Final point for this exam = total number of points/10 + 1

1. **Lenses (24 points)**

A focussing lens changes the shape of the wavefront from a plane wave to a spherical wave or vice versa. Here, we consider a lens placed in air with a focal length F (defined as the distance from the (center of the) lens to the point at which the spherical wave converges) of 400 mm, a (central) thickness D of 1 cm, with a refractive index of 1.5.



a) How does the phase of an optical wave vary across the wave front of a plane wave? And of a spherical wave? **(2 points)**

b) What is the relation between the refractive index of a material and the phase velocity of an EM wave? How does the refractive index depend on the electric permittivity and magnetic susceptibility? **(2 points)**

c) State Fermat's principle. **(2 points)**

d) Give Snell's law of refraction. **(2 points)**

e) Explain briefly what is meant with "dispersion". **(2 points)**

f) Give the definition of the optical path length. Calculate the optical path length from the entrace surface of the lens to the focal point for a ray going through the center of the lens. **(2 points)**

g) Derive that the thickness of the lens can be parametrized as $D - A \cdot r^2$ and calculate A. Assume that the entrance of the lens is flat. Hint: use that $\sqrt{1+2x} \simeq 1 + x$, and ignore terms d^n with $n > 1$. Explain your quantities with a drawing. **(6 points)**

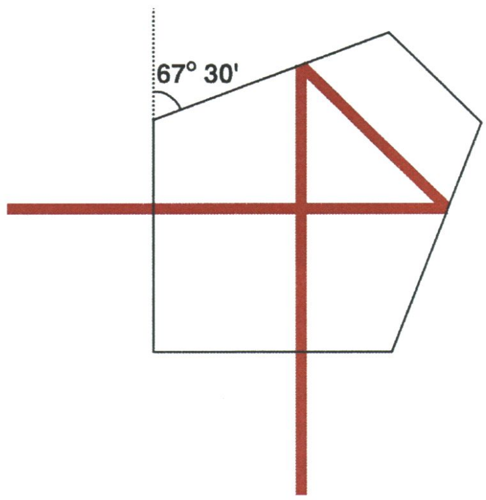
h) Use Snell's law to show that a ray that enters the lens 1 cm from the centerline crosses the focal point. If you couldn't solve g), use Snell's law to calculate A. **(4 points)**

i) If the refractive index for blue light is 1% larger than for green light, will the lens focus the blue light *before* or *behind* the green light? **(2 points)**

2. **Fiber Communication** **(21 points)**

a) What is meant by "total internal reflection"? Under what condition does it occur? **(2 points)**

b) Consider the (symmetric) penta-prism in the figure below. When suspended in air, find out for which refractive indices it would work. **(4 points)**



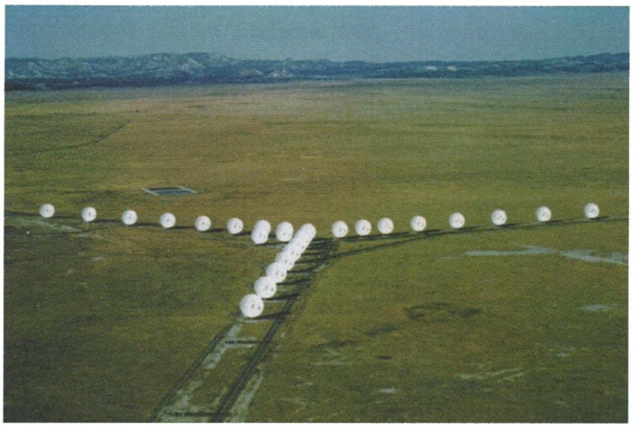
c) Give the definitions for the phase and group velocities. Can they be different, and if so, under what condition(s)? **(3 points)**

d) For communication over large distances a train of short light pulses is sent through an optical fiber. Explain how dispersion limits the useful length of the fiber. **(4 points)**

e) What is a standing wave? How can you create one? **(2 points)**

f) Light for fiber-communication is generally generated by a laser, which consists of a light amplifying medium placed inside a Fabry-Perot etalon with a very high coefficient of finesse. Explain why this results in a beam of light with a very large coherence length. **(6 points)**

3. **Radio Telescope Array (26 points)**
We will consider the detection of radio waves from a far away galaxy using a set of dish-shaped radio detectors. The figure below shows the Very Large Array (VLA) as an example of such a setup.



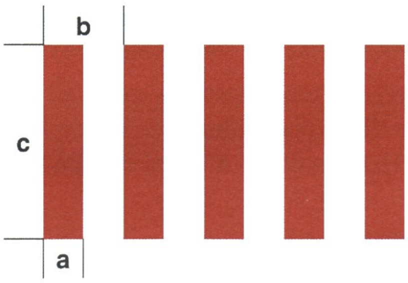
a) Give an equation for the electric field (in 3D) of a plane wave traveling in vacuum with a wavelength λ. **(3 points)**

b) What is the relation between the electric and magnetic field amplitude of an electromagnetic wave (in vacuum)? How are the electric and magnetic field vectors oriented with respect to each other and the wave vector? **(3 points)**

c) What are the definitions of the Poynting vector and irradiance? **(2 points)**

d) What is the difference between Fraunhofer and Fresnel diffraction? **(2 points)**

e) Give the equation for the 2D Fraunhofer interference pattern for the 2D slit configuration shown below (colored areas represent holes in an otherwise opaque screen). Assume the wavelength is λ. **(6 points)**



f) Briefly explain what the Airy disk is. **(2 points)**

g) Argue (don't calculate!) which dimension of the VLA, the dish diameter, the separation between the dishes, or the size of the entire array of dishes, determines the smallest object that can be "seen" (or resolved) by it? **(4 points)**

h) Will the detection of radio waves from a distant source suffer more or less from Rayleight scattering in the Earth's atmosphere than optical detection? Explain. **(4 points)**